Forces on Convex Bodies in Free Molecular Flow

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In this paper general expressions are derived for the aerodynamic forces on a body when the momentum transferred to the surface depends on the local surface inclination only. Applied to free molecular flow conditions, an added advantage of the technique is to separate the derivation of the forces from the uncertain physics of the gas-surface interaction. The general expressions obtained for the forces contain constants of integration which are determined by reference to the particular body shape. As an example, these constants are evaluated for any axisymmetric segment, and the expressions are used to find the drag of slowly tumbling axisymmetric bodies for a range of conditions, including typical Earth satellite conditions.

I. Introduction

IN a previous paper[1] it has been shown that if the pressure on the surface of a body is given by Newtonian theory (i.e. $C_p \alpha \cos^2\delta$), general analytic expressions for the forces on the body can be obtained as solutions to Poisson's equation. In this paper similar solutions are obtained when the streamwise and transverse momentum transfer at the surface is given by

$$M_v = \sum N_p \cos^p \delta \tag{1}$$

$$M_t = \tan\delta \sum T_p \cos^p \delta \tag{2}$$

where δ is the surface inclination angle as shown in Fig. 1, p is a positive integer or zero, and N_p and T_p are constants.†

The analysis is applied, in particular, to finding the forces on convex bodies in free molecular flow. In this application, the preferred direction v is in the stream or flow direction, and it is anticipated that this would normally be so. It should be noted however, that the theory is applicable to cases where the direction of v is not streamwise. For example, in finding the forces due to radiation pressure from the Sun, v would need to be aligned with the Sun's rays.

In free molecular flow the momentum incident on a surface element is given by[2,3]

$$M_v = \frac{\rho V^2 dS}{2s} \{ (1 + \frac{1}{2s^2}) \sigma (1 + \operatorname{erf} \sigma) + \frac{1}{\pi^{1/2}} \exp(-\sigma^2) \} \tag{3}$$

$$M_t = \rho V^2 dS \sin\delta (1 + \operatorname{erf}\sigma)/4s^2 \tag{4}$$

where V is the mass velocity of the gas, s is the ratio of V to the most probable molecular speed, and $\sigma = s\cos\delta$. For $s=1$ and 6, M_v/M_0 and M_t/M_0 are shown in Fig. 2, where M_0 is defined as the momentum flux through an area dS normal to the stream, i.e.

$$M_o = M_v(\delta=0) = \frac{\rho V^2 dS}{2s} \{ (s + \frac{1}{2s}) \times (1 + \operatorname{erf} s) + \pi^{-1/2} \exp(-s^2) \}$$

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†Recently the Eqs. for axisymmetric bodies given in my early paper[8] have been generalized in a similar manner to that used here, by A. I. Bunimovich.[9] As he correctly states, the expression for C_L in my 1969 paper is in error. The final index in Eq. (5) for C_L should be 2 and not 3.

To represent the M_v and M_t of Eqs. (3) and (4) by Eqs. (1) and (2), appropriate values have to be assigned to N_p and T_p. For example, when $s=1$ it is sufficient to specify only $N_0 - N_2$ and $T_1 - T_3$ as $0.2M_0$, $0.5M_0$, $0.3M_0$, $0.17M_0$, $0.17M_0$ and $-0.05M_0$ respectively. For $s=6$, using $N_0 - N_2$ only gives errors in M_v of up to $0.07M_0$. This error is unacceptable, as

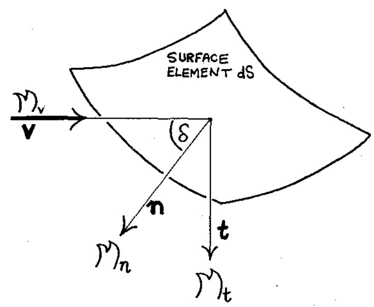

Fig. 1 Streamwise (v), normal (n), and transverse (t) vectors at the surface.

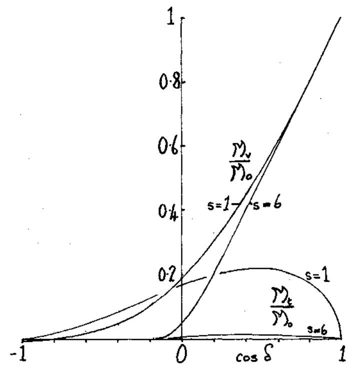

Fig. 2 Streamwise and transverse momentum of incident molecules.

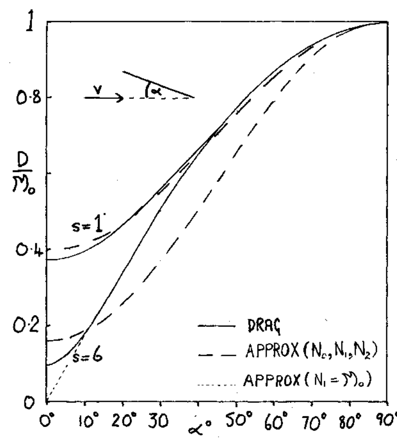

Fig. 3 Drag of a flat plate.

can be seen (Fig. 3) by calculating the simple but critical case of the drag of a flat plate at incidence. To improve the estimate for $s=6$, either more terms can be included in the series for M_v, or more simply since nearly all the contribution to the forces comes from the front surface of the body, the region of integration can be restricted to $\cos\delta \geqslant 0$. For this latter case putting $N_1 = M_0$ gives the much improved approximation also shown in Fig. 3. This approximation is used in Sec. IV.

For the reflected molecules, simple models such as the specular-diffuse approximation are generally inadequate to describe the reflection processes[3] and more realistic models are required. It is suggested in Ref. 3 that a better representation is given by series in $\cos p\delta$ and $\sin\delta\cos p\delta$, which it can easily be shown transform exactly into Eqs. (1) and (2). Thus theoretical coefficients derived in Ref. 3 are of direct application here. However, with the existing incomplete state of the theory, it is preferable to derive the coefficients from reliable experimental data if available. In Fig. 4 some recent experimental data[4] for M_v/M_0 is shown. Also shown are curves passing close to the data for which N_0-N_3 have values 0, 0.2 M_0, -0.2 M_0, and -0.2 M_0 respectively for 5eV incident energy, and 0, 0.36 M_0, $-0.777\,M_0$, and 0.392 M_0 for 200 eV incident energy. The total momentum transfer at the surface is obtained by subtracting the coefficients of N_p and T_p for the reflected molecules from the incident molecules.

In this paper the derivation of values for N_p and T_p is not of primary concern, for the expressions for the forces are derived with these as parameters and the forces may be evaluated using the most accurate values available at the time.

II. General Expressions for the Forces on Convex Bodies

The force on a body due to momentum transfer at the surface is given by

$$F = \int_s (M_v \boldsymbol{v} - M_t\, \boldsymbol{t})\; dS \tag{5}$$

where S is the surface area of the body and $\boldsymbol{t}$ is the unit transverse vector shown in Fig. 1 to be given by

$$\boldsymbol{t} = \boldsymbol{n}\ \text{cosec}\ \delta + \boldsymbol{v}\ \cot\ \delta \tag{6}$$

with $\cos\delta = -\boldsymbol{v}\cdot\boldsymbol{n}$. The drag, lift, and side force are defined as the components of force along with axes $\boldsymbol{v}, \boldsymbol{l}$, and $\boldsymbol{s}$, whence from Eqs. (1, 2, 5 and 6)

$$D = F\cdot \boldsymbol{v} = \int_s M_v \mathrm{d}S = \sum N_p\, D_p \tag{7}$$

where

$$D_p = (-1)^p \int_s (\boldsymbol{v}\cdot\boldsymbol{n})^p \mathrm{d}S \tag{8}$$

$$L = F\cdot \boldsymbol{l} = \int_s M_t \text{cosec}\delta(\boldsymbol{n}\cdot\boldsymbol{l})\mathrm{d}S = \sum T_p L_p \tag{9}$$

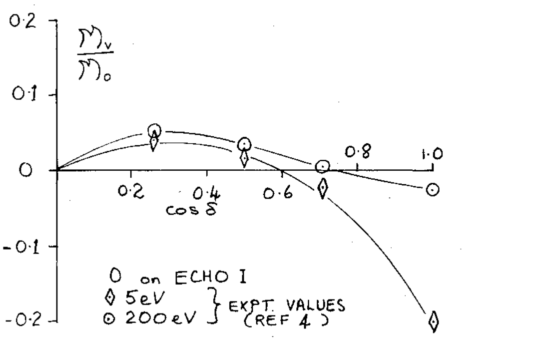

Fig. 4 Ratio of reflected streamwise momentum to incident momentum.

where

$$L_p = (-1)^p \int_s (\boldsymbol{v}\cdot\boldsymbol{n})^{p-1}(\boldsymbol{l}\cdot\boldsymbol{n})\mathrm{d}S \tag{10}$$

and

$$Si = F\cdot \boldsymbol{s} = \int_s M_t \text{cosec}\delta(\boldsymbol{n}\cdot\boldsymbol{s})\mathrm{d}S = \sum T_p S_p \tag{11}$$

where

$$S_p = (-1)^p \int_s (\boldsymbol{v}\cdot\boldsymbol{n})^{p-1}(\boldsymbol{s}\cdot\boldsymbol{n})\mathrm{d}S \tag{12}$$

We show that the integrals L_p and S_p can be expressed in terms of D_p, and thus the determination of the forces on a body for a range of incidence (α) and yaw (ψ) can be reduced simply to determining the values of D_p.

To find the relationships between L_p, S_p, and D_p, define body axes ($\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$) as shown in Fig. 5, which are related to the wind axes by

$$\boldsymbol{v} = \boldsymbol{e}_1\cos\alpha\cos\psi + \boldsymbol{e}_2\sin\alpha\cos\psi - \boldsymbol{e}_3\sin\psi \tag{13}$$

$$\boldsymbol{l} = -\boldsymbol{e}_1\sin\alpha + \boldsymbol{e}_2\cos\ \alpha \tag{14}$$

$$\boldsymbol{s} = \boldsymbol{e}_1\cos\alpha\ \sin\psi + \boldsymbol{e}_2\sin\alpha\ \sin\psi + \boldsymbol{e}_3\cos\psi \tag{15}$$

Differentiating Eq. (8) with respect to α, gives

$$D_{p\alpha} = (-1)^p p \int_s (\boldsymbol{v}\cdot\boldsymbol{n})^{p-1}(\boldsymbol{v}_\alpha\cdot\boldsymbol{n})\mathrm{d}S = p\, L_p\cos\psi \tag{16}$$

Similarly

$$D_{p\psi} = (-1)^p p \int_s (\boldsymbol{v}\cdot\boldsymbol{n})^{p-1}(\boldsymbol{v}_\psi\cdot\boldsymbol{n})\mathrm{d}S = -p\, S_p \tag{17}$$

giving L_p and S_p in terms of D_p as required.

Turning now to the determination of D_p, we note first that D_0 and D_1 are given by

$$D_0 = \int_s \mathrm{d}S = S \tag{18}$$

$$D_1 = -\int_s (\boldsymbol{v}\cdot\boldsymbol{n})\mathrm{d}S = A_p \tag{19}$$

where A_p is the projected area of the body surface in the flow direction, which is zero if the surface is "complete" or "closed." For $p>1$ it can be shown by substitution that D_p satisfies

$$D_{p\alpha\alpha}\sec^2\psi + D_{p\psi\psi} - D_{p\psi}\tan\psi + p(p+1)D_p = p(p-1)D_{p-2} \tag{20}$$

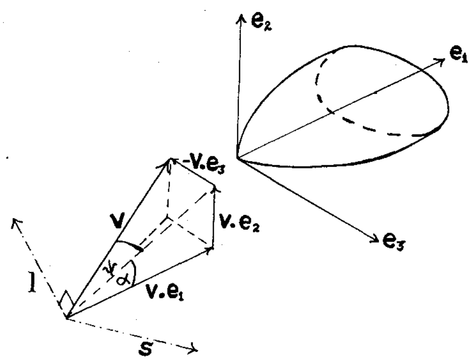

Fig. 5 Angles of incidence (α) and yaw (ψ).

When $p=3$ this equation is similar to that derived for the forces in Newtonian theory.[5] The equation is a degenerate form of Poisson's equation[1] whose solution can be expressed[6] as the sum of a complementary function F_p and a particular integral I_p where

$$F_p=\sum_{q=0}^{p}(K_{pq}\sin q\alpha+K'_{pq}\cos q\alpha)P_p^q(\sin\psi) \tag{21}$$

with P_p^q an Associated Legendre Function of the first kind and I_2, I_3, I_4 particular integrals of

$$D_{2\alpha\alpha}\sec^2\psi+D_{2\psi\psi}-D_{2\psi}\tan\psi+6D_2=2S \tag{22}$$

$$D_{3\alpha\alpha}\sec^2\psi+D_{3\psi\psi}-D_{3\psi}\tan\psi+12D_3=6A_p \tag{23}$$

$$D_{4\alpha\alpha}\sec^2\psi+D_{4\psi\psi}-D_{4\psi}\tan\psi+20D_4=12D_2 \tag{24}$$

Several solutions to Eq. (23) are presented in Ref. 1. Generalizing one of these solutions, we find that if D_{p-2} is of the form

$$D_{p-2}=[p(p+1)-a(a+1)+(b^2-c^2)\sec^2\psi]\,{}^{\sin}_{\cos}(c\alpha)P_a{}^b \tag{25}$$

where a, b, and c are constants, then the value of I_p is given from Eq. (20) by

$$I_p=p(p-1)\,{}^{\sin}_{\cos}(c\alpha)P_a^b \tag{26}$$

Of particular interest is the case $b=c$ for this includes all the terms in the complementary function. Thus by successive use of this solution we find

$$D_2=F_2+I_2(S) \tag{27}$$

$$D_3=F_3+I_3(A_p) \tag{28}$$

$$D_4=F_4+6F_2/7+I_4(S) \tag{29}$$

$$D_5=F_5+10F_3/9+I_5(A_p) \tag{30}$$

and in general

$$D_p=\sum_{\tau=0}^{p/2-1}\frac{p!(p-r)!(2p-4r+1)!F_{p-2r}}{(p-2r)!(p-2r)!(2p-2r+1)!r!}+I_p \tag{31}$$

As the value of F_p is defined by Eq. (21), it remains to find the values of I_p, which for p even depend on S and for p odd depend on A_p. In many cases S and A_p can conveniently be expanded as a series in the orthogonal functions $\sin c\alpha P_a^b$, when successive use of the general solution of Eq. (25) defines the particular integral for all p.

In general, the region of body surface S for which the forces are to be evaluated will be the whole body surface or a fixed portion of it (e.g., the body surface minus the base region). For a fixed surface region we have immediately

$$S=\text{constant} \tag{32}$$

and

$$A_p=A\sin\alpha\cos\psi+B\cos\alpha\cos\psi+C\sin\psi \tag{33}$$

where A, B and C are constants representing the projected area of S in the directions e_1, e_2, and e_3, respectively, (e.g., the plan, base, and side areas). Then, from Eqs. (22-24) and similar equations for $p>4$, by using the general solution of Eqs. (25) and (26), we find for p even

$$I_p=S/(p+1) \tag{34}$$

and for p odd

$$I_p=3A_p/(p+2) \tag{35}$$

Thus, for bodies with S a constant region, Eqs. (7, 9, 11, 16, 17, 21, and 31-35) give complete analytic solutions for the forces, where the particular body shape is identified by the constants of integration which occur in Eq. (21) for F_p. The expressions for $D_0(\alpha,\psi)$-$D_3(\alpha,\psi)$ in terms of the constants of integration become

$$D_0=S \tag{36}$$

$$D_1=A_p \tag{37}$$

$$D_2=\tfrac{1}{2}K'_{20}(3\sin^2\psi-1)+3(K_{21}\sin\alpha+K'_{21}\cos\alpha)\sin\psi\cos\psi+3(K_{22}\sin 2\alpha+K'_{22}\cos 2\alpha)\cos^2\psi+S/3 \tag{38}$$

$$D_3=\tfrac{1}{2}K'_{30}(5\sin^3\psi-3\sin\psi)+1.5(K_{31}\sin\alpha+K'_{31}\cos\alpha)\times(5\sin^2\psi-1)\cos\psi+15(K_{32}\sin 2\alpha+K'_{32}\cos 2\alpha)\sin\psi\cos^2\psi+15(K_{33}\sin 3\alpha+K'_{33}\cos 3\alpha)\cos^3\psi+3A_p/5 \tag{39}$$

where L_0 to L_3 and S_0 to S_3 are obtained by differentiating with respect to α and ψ, respectively, in accordinace with Eqs. (16) and (17).

For most bodies of interest, the previous equations simplify further; for example, if the area S is the whole body surface, then $A_p=0$. Also, if the body has a vertical plane of symmetry D_p is an even function of ψ and

$$K_{21}=K'_{21}=K'_{30}=K_{32}=K'_{32}=0 \tag{40}$$

and if there is a horizontal plane of symmetry

$$K_{21}=K_{22}=K_{31}=K_{32}=K_{33}=0 \tag{41}$$

Evaluation of the remaining constants of integration requires calculation of the forces at some convenient orientations. This is demonstrated for particular examples in Sec. III.

III. Some Results for Axisymmetric Segments and Rotating Bodies

To find the forces on a body by the method developed here, it is necessary to determine the values of D_p, L_p, and S_p in the expressions of Eqs. (7, 9, and 11) for sufficient terms such that Eqs. (1) and (2) give an adequate description of the mean streamwise and transverse momentum transfer of the molecules. For most gas-surface interactions, four terms are sufficient to describe M_v and M_t, so as an example we derive D_0-D_3 (and the appropriate L_p and S_p values) for any axisymmetric segment (see Fig. 6). In this case, the constants of integration are based on the calculated values of D_p and L_p at $\alpha=\psi=0$. These are

$$D_p(0,0)=\phi_0\int_{x_o}^{x_b}\left(\frac{R'^2}{1+R'^2}\right)^{(p-1)/2}RR'\,dx \tag{42}$$

$$L_p(0,0)=2\sin(\phi_0/2)\int_{x_o}^{x_b}\left(\frac{R'^2}{1+R'^2}\right)^{(p-1)/2}R\,dx \tag{43}$$

where ϕ_0 is the included angle of the segment, that is $\phi_0=2\pi$ for an axisymmetric body.

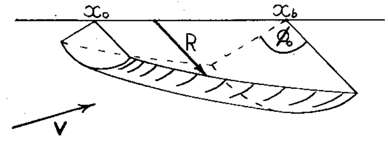

Fig. 6 An axisymmetric segment.

These integrals must be evaluated for the particular axisymmetric profile $R=R(x)$. For a cone segment of semiangle θ_c when $R=x\tan\theta_c$ they become

$$D_p(0,0)=\tfrac{1}{2}\phi_0(\sin\theta_c)^{p-1}(R_b^2-R_0^2) \quad (44)$$

$$L_p(0,0)=\sin(\phi_0/2)(\sin\theta_c)^{p-1}\cot\theta_c(R_b^2-R_0^2) \quad (45)$$

From Eqs. (36) and (42) we have

$$D_0=S=D_0\ (0,0) \quad (46)$$

and from Eqs. (33) and (37), noting the symmetry about $\psi=0$, we have

$$D_1=A_p=A\ \sin\alpha\cos\psi+B\ \cos\alpha\ \cos\psi \quad (47)$$

where

$$A=2\ \sin(\phi_0/2)\int_{x_0}^{x_b} R\ dx=L_1(0,0) \quad (48)$$

$$B=\tfrac{1}{2}\phi_0(R_b^2-R_0^2)=D_1(0,0) \quad (49)$$

Expressions for L_1 and S_1 are obtained from D_1 using Eqs. (16) and (17) i.e.,

$$L_1=A\ \cos\alpha-B\ \sin\alpha \quad (50)$$

$$S_1=A\ \sin\alpha\ \sin\psi+B\ \cos\alpha\ \sin\psi \quad (51)$$

For D_2, Eq. (38) gives

$$D_2(\alpha,\psi)=\tfrac{1}{2}K'_{20}(3\sin^2\psi-1)+3(K_{22}\sin 2\alpha+K'_{22}\cos 2\alpha)\cos^2\psi+S/3 \quad (52)$$

with

$$L_2(\alpha,\psi)=3\ \cos\psi(K_{22}\cos 2\alpha-K'_{22}\sin 2\alpha) \quad (53)$$

$$S_2(\alpha,\psi)=3\ \sin\psi\ \cos\psi(K_{22}\sin 2\alpha+K'_{22}\cos 2\alpha-\tfrac{1}{2}K'_{20}) \quad (54)$$

Substituting $\alpha=\psi=0$ in Eqs. (52) and (53) and using also

$$D_2(90,0)=\tfrac{1}{2}(\phi_o+\sin\phi_0)\int_{x_o}^{x_b}\frac{Rdx}{(1+R'^2)^{1/2}}$$

$$=\tfrac{1}{2}(1+\frac{\sin\phi_0}{\phi_0})\ (S-D_2(0,0)) \quad (55)$$

the constants of integration can be expressed as

$$K'_{20}=\tfrac{1}{2}[S-D_2(0,0)]\ [1-(\sin\phi_0)/\phi_o]-S/3 \quad (56)$$

$$K_{22}=\tfrac{1}{3}L_2(0,0) \quad (57)$$

$$K'_{22}=-[S-D_2(0,0)]\ [3+(\sin\phi_o)/\phi_0]/12+S/6 \quad (58)$$

Similarly, when $p=3$, we can solve Eq. (23) with D_1 given by Eq. (47) to obtain (as in Ref. 1)

$$D_3=\kappa\ \sin\alpha+\gamma\ \cos\alpha+\lambda\ \sin 3\alpha+\mu\ \cos 3\alpha \quad (59)$$

$$L_3=(\tfrac{1}{3}\kappa\cos\alpha-\tfrac{1}{3}\gamma\sin\alpha+\lambda\ \cos 3\alpha-\mu\sin 3\alpha)\sec\psi \quad (60)$$

$$S_3=-\tfrac{1}{3}\kappa_\psi\sin\alpha-\tfrac{1}{3}\gamma_\psi\cos\alpha+(\lambda_0\sin 3\alpha+\mu_0\cos 3\alpha)\sin\psi\ \cos^2\psi \quad (61)$$

where

$$\kappa(\psi)=\cos\psi\ [3A\ \sin^2\psi+\kappa_0(5\ \cos^2\psi-4)] \quad (62)$$

$$\gamma(\psi)=\cos\psi\ [3B\ \sin^2\psi+\gamma_0(5\ \cos^2\psi-4)] \quad (63)$$

$$\lambda(\psi)=\lambda_0\cos^3\psi \quad (64)$$

$$\mu(\psi)=\mu_0\cos^3\psi \quad (65)$$

with

$$\kappa_0=[(1-\cos\phi_0)L_3(0,0)+(5+\cos\phi_0)A]/8 \quad (66)$$

$$\gamma_0=[3(\phi_0-\sin\phi_0)D_3(0,0)+3(\phi_0+\sin\phi_0)B]/8\phi_0 \quad (67)$$

$$\lambda_0=[(23+\cos\phi_0)L_3(0,0)-(5+\cos\phi_0)A]/24 \quad (68)$$

$$\mu_0=[(5\phi_0+3\ \sin\phi_0)D_3(0,0)-3(\phi_0+\sin\phi_0)B]/8\phi_0 \quad (69)$$

Then for any axisymmetric segment, the drag is given as a function of α and ψ from Eq. (7) and the values of D_p by

$$\begin{aligned}D(\alpha,\psi)=&\{N_0S+N_2[S/3+\tfrac{1}{2}K'_{20}(3\ \sin^2\psi-1)]\}\\&+\{N_1A+3N_3A\sin^2\psi+N_3\kappa_0(5\cos^2\psi-4)\}\cos\psi\sin\alpha\\&+\{N_1B+3N_3B\sin^2\psi+N_3\gamma_0(5\cos^2\psi-4)\}\cos\psi\cos\alpha\\&+3K_{22}N_2\cos^2\psi\sin 2\alpha+3K'_{22}N_2\cos^2\psi\cos 2\alpha\\&+N_3\lambda_0\cos^3\psi\sin 3\alpha+N_3\mu_0\cos^3\psi\ \cos 3\alpha\end{aligned} \quad (70)$$

where $K'_{20},K_{22},K'_{22},\ \kappa_0,\gamma_0,\ \lambda_0,\ \mu_0$ are given in terms of D_p (0,0) and L_p (0,0) by Eqs. (56-58) and (66-69), respectively. The lift and side force are given similarly from L_p and S_p using Eqs. (9) and (11).

For an unyawed axisymmetric segment, or for an axisymmetric body where the angle α is redefined to be the angle between the axis of symmetry and the wind direction (v), Eq. (70) becomes on putting $\psi=0$

$$\begin{aligned}D(\alpha)=&(N_0S+N_2S/3-N_2K'_{20}/2)+(N_1A+N_3\kappa_0)\sin\alpha\\&+(N_1B+N_3\gamma_0)\cos\alpha+3N_2K_{22}\sin 2\alpha+3N_2K'_{22}\cos 2\alpha\\&+N_3\lambda_0\sin 3\alpha+N_3\mu_0\cos 3\alpha\end{aligned} \quad (71)$$

Applying axisymmetric symmetry conditions and $\phi_0=2\pi$ to the parameters, this equation becomes simply

$$\begin{aligned}D(\alpha)=&(N_0S+N_2S/4+N_2D_2(0,0)/4)\\&+(N_1B+3N_3B/8+3N_3D_3(0,0)/8)\cos\alpha\\&+N_2(3D_2(0,0)/4-S/4)\cos 2\alpha\\&+N_3(5D_3(0,0)/8-3B/8)\cos 3\alpha\end{aligned} \quad (72)$$

where $S=D_0(0,0)$, $B=D_1$ (0,0), and D_p (0,0) is given by Eq. (42) with $\phi_0=2\pi$. The lift may similarly be derived from Eqs. (50, 53, and 60) to be

$$\begin{aligned}L(\alpha)=&-(T_1B+T_3B/8+T_3D_3(0,0)/8)\sin\alpha\\&+T_2(S/4-3D_2(0,0)/4)\sin 2\alpha\\&+T_3(3B/8-5D_3(0,0)/8)\sin 3\alpha\end{aligned} \quad (73)$$

These equations give the forces on any axisymmetric body at any incidence when the momentum transfer at the surface may be described by Eqs. (1) and (2) with $p\leq 3$.

If the forces on a body are available for any orientation, then we may consider the average forces on the body for slow rotations or oscillations. In free molecular flow, as the forces and moments must be expected to be small compared with the inertia of the body, we may expect an unconstrained body to rotate or tumble with nearly constant angular velocity about some axis. Let this axis of rotation make angles χ and ζ with the axes of symmetry (e_1) and the wind direction (V) respectively. Then the average drag of a tumbling body is given by

$$\langle D\rangle = \frac{1}{2\pi}\int_0^{2\pi} D(\alpha)\ \mathrm{d}\beta \qquad (74)$$

where

$$\cos\alpha = \cos\chi\ \cos\zeta + \sin\chi\ \sin\zeta\ \cos\beta \qquad (75)$$

Thus, using Eq. (72), the drag of any slowly tumbling axisymmetric body is given by

$$\begin{aligned}\langle D\rangle = &[N_0S+N_2S/2-N_2D_2(0,0)/2]\\ &+[N_1B+3N_3B/2-3N_3D_3(0,0)/2]\cos\chi\ \cos\zeta\\ &+N_2[3D_2(0,0)/2-S/2]\cos^2\chi\cos^2\zeta\\ &+N_3[5D_3(0,0)/2-3B/2]\cos^3\chi\cos^3\zeta\\ &+\sin^2\chi\ \sin^2\zeta(N_2[3D_2(0,0)/4-S/4]\\ &+3N_3[5D_3(0,0)/4-3B/4]\cos\chi\ \cos\zeta)\end{aligned} \qquad (76)$$

Consider, for example, the rocket shaped body shown in Fig. 7 rotating about the axis indicated $(\chi=\zeta=90°)$. For this body $S=15.92$ $B=0$, D_2 $(0,0)=1.34$, and D_3 $(0,0)=-0.88$ giving

$$\langle D\rangle = 15.92\ N_0 + 4.32\ N_2 \qquad (77)$$

Then, the incident molecules in a free molecular flow with $s=1$ give a drag of 4.48 M_0 for this rotating body. At $S=6$, which is typical of satellite conditions if 0.08 + 0.5 cosδ +0.42 cos$^2\delta$ is used as a description of M_v/M_0, the drag is given by 3.1 M_0. However, to obtain a reliable value, a better approximation to M_v is required. In the following section, by restricting the region of integration to cos$\delta \geq 0$, a more accurate value is found to be 3.39 M_0.

IV. Forces on Forward Facing Surfaces

For $s \gg 1$, nearly all the contribution to the forces acting on the body comes from the streamwise facing surface (cos$\delta \geq 0$).

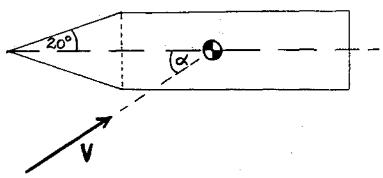

Fig. 7 A rocket-shaped body.

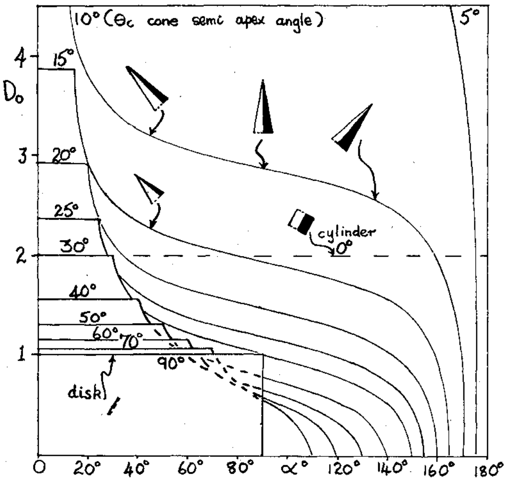

Fig. 8 Wetted area (D_0) for cones of unit base area.

Hence, the significant error involved in using only 3 or 4 terms of Eq. (1) when s is large, may be reduced by restricting the region of integration to cos$\delta \geq 0$. The disadvantage of this approach is that the surface of intergration varies with α and ψ and new values of I_p must be derived for the particular expressions of $S(\alpha,\psi)$ and A_p (α,ψ). However, for axisymmetric bodies, we can adapt the previously derived equations by describing the body as a number of cone sections. Consider, for example, the rocket shape shown in Fig. 7. This can

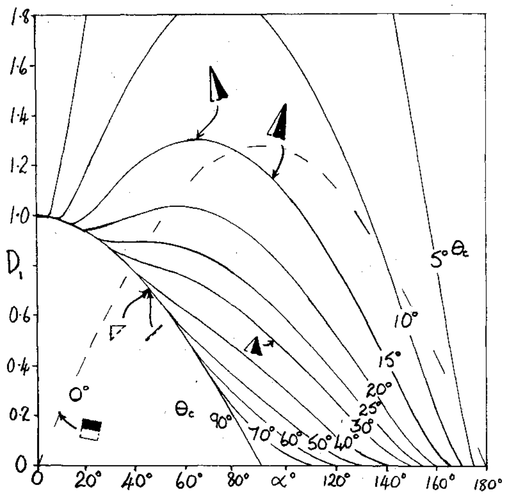

Fig. 9 Streamwise projected area (D_1) for cones of unit base area.

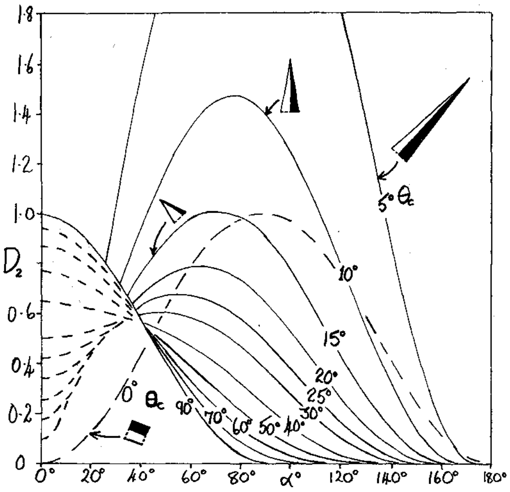

Fig. 10 D_2 for cones of unit base area.

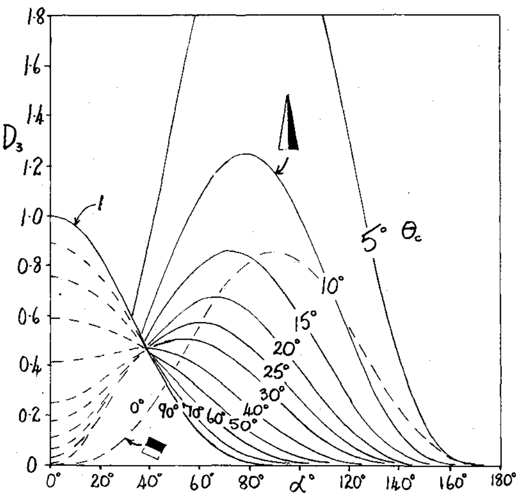

Fig. 11 D_3 for cones of unit base area.

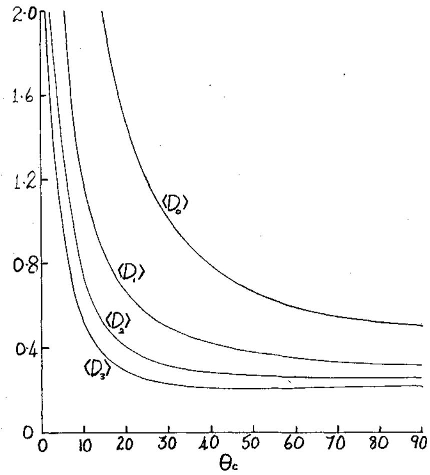

Fig. 12 Average values of D_0, D_1, D_2, and D_3 from Figs. 8-11.

be considered as three cone sections with θ_c equal to 20°, 0°, and 90°. The forces on this body may be obtained by adding their contributions, that is for the body of Fig. 7, where $D_p(\alpha,\theta_c)$ represents the cone value

$$D_p = D_p(\alpha, 20°) + 3D_p(\alpha, 0°) + D_p(180° - \alpha, 90°) \quad (78)$$

The cone D_p values at any incidence α, can be obtained from the axisymmetric segment values of the previous section by putting ϕ_0 equal to 2π for $\alpha \le \theta_c$, $2\cos^{-1}(\tan\theta_c/\tan\alpha)$ for $\theta_c \le \alpha \le \pi - \theta_c$, and 0 for $\alpha \ge \pi - \theta_c$. That is $D_p(\alpha,\theta_c)$ is given by Eq. (71), where the parameters are given by Eqs. (44, 45, 48, 49, 56-58, and 66-69), with the appropriate values of ϕ_0. The values of D_0-D_4 for cones with unit base area are shown in Figs. 8-11. Also shown are values for a cylinder of unit cross-sectional area and length equal to the diameter. The drag of the body shown in Fig. 7 may then easily be calculated for any α using Eq. (78) and Figs. 8-11. For example, at 30° incidence

$$D = 8.09N_0 + 2.88N_1 + 1.34N_2 + 0.69N_3 \quad (79)$$

The summation method used to calculate the drag of an axisymmetric body at fixed α, may also be applied to a rotating body. For the case $\chi = \zeta = 90°$, the value of $\langle D_p(\theta_c)\rangle$ is the average value of $D_p(\alpha)$, shown in Figs. 8-11. This is shown plotted in Fig. 12, where the drag of the body in Fig. 7 rotating about the axis shown is given by

$$\langle D\rangle = 7.96\,N_0 + 3.39\,N_1 + 2.16\,N_2 + 1.58\,N_3 \quad (80)$$

To obtain the drag associated with the incident molecules when s is large, we put $N_1 = M_0$ in Eq. (80) to give $\langle D\rangle = 3.39\,M_0$.

V. Conclusions

Techniques which have been developed[1,7] to find the forces on bodies when the surface pressure is given by Newtonian theory ($C_p\alpha\cos^2\delta$), are generalized to find the forces on bodies when the local pressure and shear forces are any function of the surface inclination. These generalized techniques are applied to the free molecular flow past convex bodies, and expressions for the forces are obtained in terms of parameters of the gas-surface interaction and functions D_n which are solutions of Poisson's equation. For rigid bodies, these solutions are analytic functions of the body incidence and yaw containing constants of integration which need to be determined from the shape of the body. For axisymmetric body segments the constants of integration are found in a simple general form.

The derived expressions for the forces in free molecular flow form a convenient way of evaluating the forces for a wide range of conditions, and act as a powerful tool for solving problems involving the forces. Of particular significance in the latter case, is the separation of the expression into body shape parameters, gas-surface interaction parameters and functions of the body orientation. An example is presented in which an expression for the average drag of any slowly tumbling axisymmetric body is derived.

When the speed ratio of the mean molecular motion to the thermal motion is large, it is found that more than 3 or 4 terms are required to represent the local variation of the surface forces with surface inclination. The algebraic complexity associated with including more terms can be avoided, however, by neglecting the relatively small thermal motion of the molecules. The surface to be considered is then the "front" surface of the body, that being the only surface exposed to the flow. A disadvantage of this flow model simplification is that the exposed surface of the body is a function of the body orientation, and D_n cannot necessarily be found analytically. It is successfully used however to find the drag of a rotating rocket shape at satellite velocities.

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